Time series analysis in Centre Borelli
Application to gait analysis

Laurent Oudre

Duke University seminar
February 16th 2024

PART I

The Centre Borelli

Giovanni Borelli

Centre Giovanni Borelli

- Giovanni Alphonso Borelli (1608 - 1679) : Italian mathematician, philosopher, astronomer, physician and physiologist.
- Holder of the chair of mathematics at the University of Pisa
- Pioneer of physiology, which is the study of the role and functioning of living organisms and their interactions with the environment. Nervous, reproductive, circulatory, respiratory, motor, digestive systems...
- His works : study of movement (father of biomechanics), organs, digestion...

The Centre Borelli

The Centre Borelli

Fusion of two labs :

- The Centre de mathématiques et de leurs applications (CMLA) : applied mathematics for the study of complex phenomena and data
- The Cognition & Action Group (CognacG) : quantification and study of human and animal behavior

Main topics of research

- Mathematical foundations and algorithms for artificial perception and learning
  - Image and video processing
  - Signal processing
  - Machine learning
  - Network science.
- Modeling, mathematical analysis and simulation of complex physical, natural and biological phenomena
  - Computational biology & molecular dynamics modeling
  - Simulation and modeling of complex physical systems
- Integrative and behavioral neurosciences in humans and animals
  - Study of the states of consciousness
  - Mental load and complex human-machine interfaces
  - Sensorimotor models and their applications
  - Developmental Trajectories & Psychiatry

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  - **Developmental Trajectories & Psychiatry**

The Signal Group

Charles Truong, postdoc

Change-point detection
Dictionary learning
Medical and industrial applications

Sylvain Combettes, postdoc

Symbolization of time series

Sam Perochon, PhD student (3A)

Multimodal signal analysis
Medical applications (SmartFlat)

Thibaut Germain, PhD student (3A)

Pattern detection and recognition
Geometric approaches
Medical applications (mouse)

Alexandre Bois, PhD student (3A)

Topological data analysis
Anomaly detection

Chrysoula Kosma, postdoc

Deep learning for time series
Industrial applications

Mona Michaud, PhD student (3A)

Neurosciences
Medical applications (gait analysis)

Quentin Laborde, PhD student (2A)

Multimodal signal analysis
Symbolization of time series
Medical applications (eye-tracking)

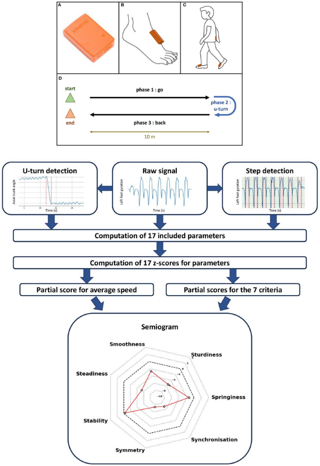
Lucas Zoroddu, PhD student (1A)

Graph signal processing
Medical applications (ECG)

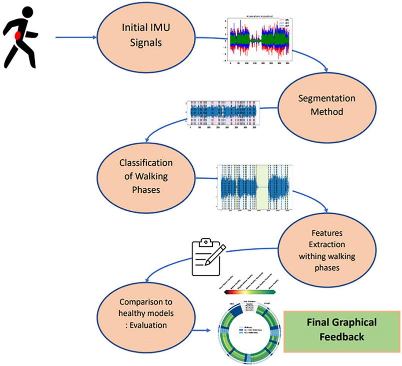
Mathieu Peyrel, predoc

Anomaly detection
Industrial applications

The Signal Group : research topics (1/5)



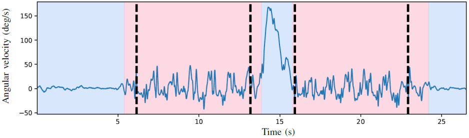
C. Voisard et al., Frontiers in Neurology, 2023



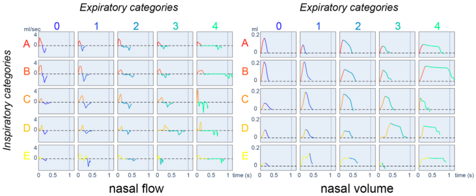
S. Jung et al., Sensors, 2023

Design of measurement and data analysis pipelines : from data acquisition and processing algorithms to information extraction and ergonomic visualization (dashboard).

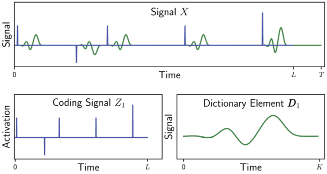
The Signal Group : research topics (2/5)



C. Truong et al., IEEE Transactions in Signal Processing, 2019



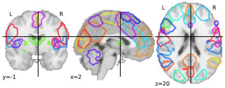
T. Germain et al., Frontiers in Physiology, 2023



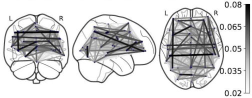
T. Moreau et al., ICML, 2018

Event detection in time series (multivariate and/or multimodal) : **pattern**, anomaly and **change-point detection**

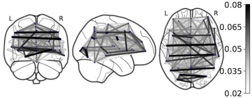
The Signal Group : research topics (3/5)



(a) Indicative Regions of Interest (ROIs) from Varoquaux et al. (2011).

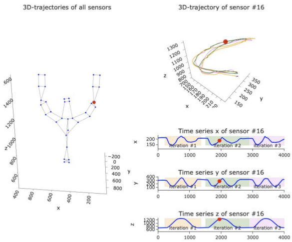


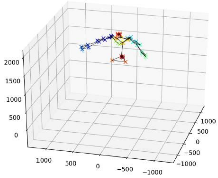
(b) ADHD subject.



(c) Healthy subject.

P. Humbert et al., JMLR, 2021

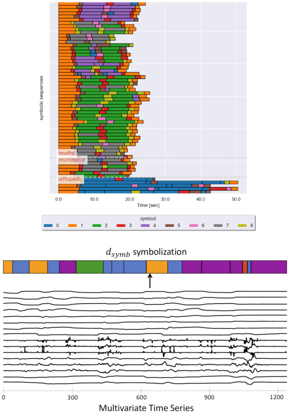




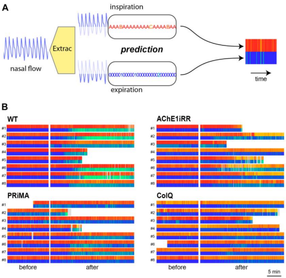
A. Mazarguil et al., Signal Processing, 2022

Tools for processing highly structured data (such as sensor networks) : graph signal processing

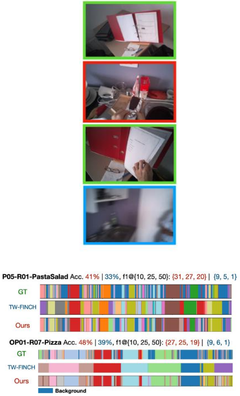
The Signal Group : research topics (4/5)



S.W. Combettes et al., ICDMW, 2023



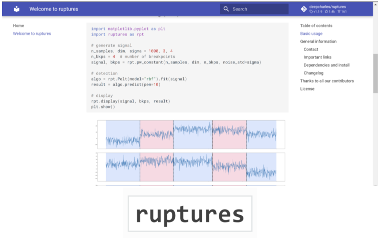
T. Germain et al., Frontiers in Physiology, 2023



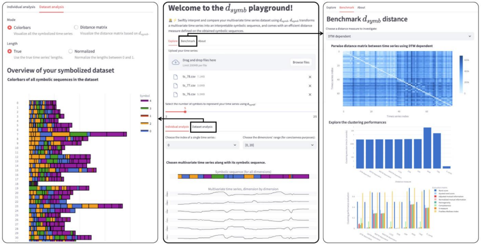
S. Perochon et al., ICASSP, 2023

Innovative, interpretable visualizations for field experts : semantic action summaries, colorbars, etc.

The Signal Group : research topics (5/5)

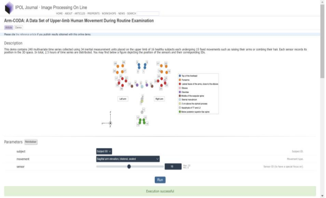


C. Truong et al., Signal Processing, 2020



(A) Dataset analysis frame (B) Individual analysis frame (C) Benchmark frame

S.W. Combettes et al., ICDE, 2024



S.W. Combettes et al., IPOL, 2024

Dissemination of results according to the standards of reproducible research : IPOL publications, Python packages (e.g. : ruptures +11M downloads), on-line demonstrators...

Today's presentation

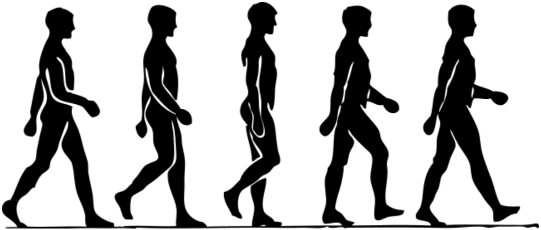
SmartCheck

- Smartcheck project : analysis of accelerometry signals for the study of human locomotion
- Example of multidisciplinary project combining fundamental and applied research
- Focus on the scientific approach and feedback on the different issues of the project
- Vision of the entire measurement chain, from the raw signals to the visualization designed for the clinician

PART II

The SmartCheck project

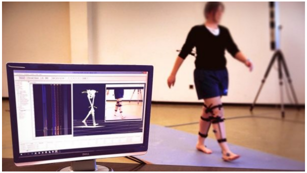
Study of locomotion



Why study the locomotion ?

- Most common dynamic human activity
- Reveals a large number of neurological, orthopedic, rheumatological disorders...
- Strong influence on daily life : risk of falling, frailty, mobility, dependence...

Study of locomotion



How to study locomotion ?

- Historically : clinical examination by the physician, functional tests, clinical questionnaires

+	Ease of execution, clinician expertise
-	Lack of precision, difficult to compare two sessions

- Platforms for studying locomotion : instrumented mats, video/optical systems

+	Very precise, extraction of a large number of parameters, objective quantification
-	High cost, difficult to implement

General principles

★ Objective quantification of locomotion

→ Use of sensors and physiological measures

★ Longitudinal follow-up and inter-individual comparison

→ Need for a fixed protocol

★ Experimentation on the field

→ Lightweight sensors and fully automatic device for consultation and routine use

★ Clean and quality data

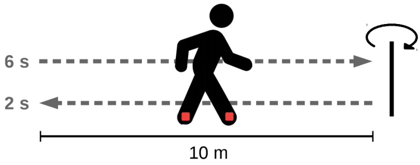
→ Control of the whole measurement chain, robust and reproducible algorithms

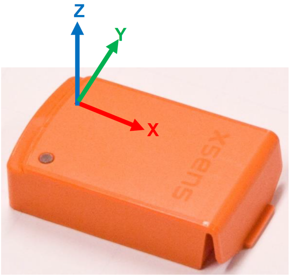
★ Willingness to capture clinician expertise

→ Clinical annotations and metadata

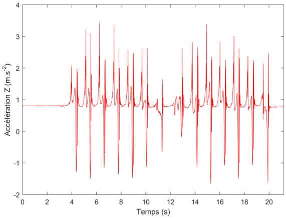
Protocol and sensors

- Comfort speed protocol : stop (6 sec), walk forward (10 m), turn around, return, stop
- Four wireless inertial units : left foot, right foot, lower back, head
- Nine signals per sensor : linear acceleration (3D), angular velocity (3D), magnetic field (3D)

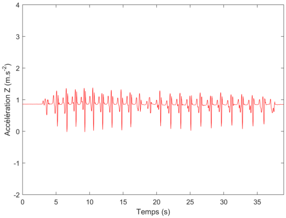




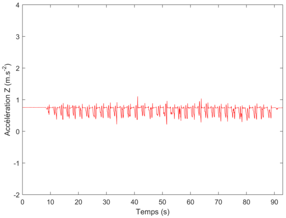
Signals



Healthy subject



Mild neurological disease



Severe neurological disease

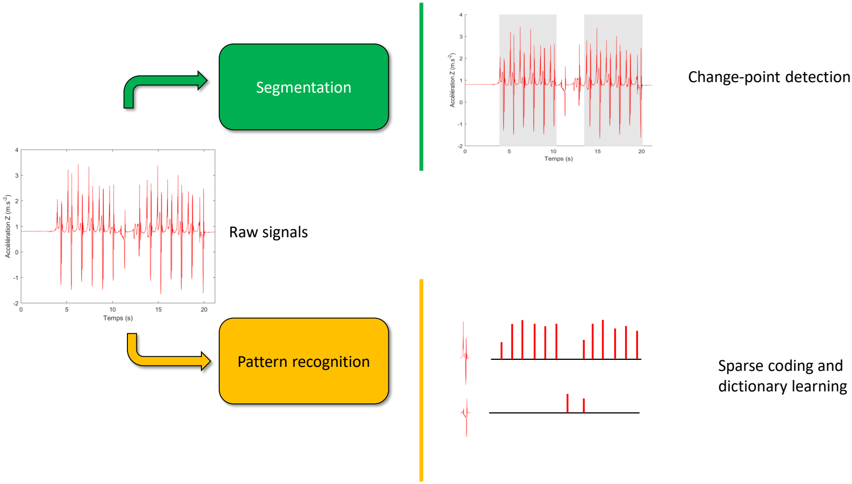
Non-stationary signals

→ How to detect the different regimes (walk, U-turn, etc.)?

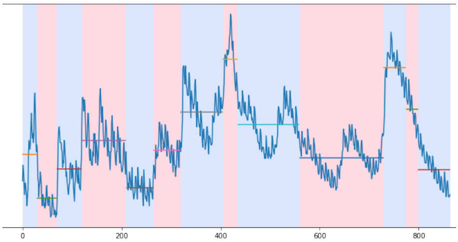
Presence of repetitive patterns

→ Can we extract them? Find the times where they occur?

Scientific questions



Problem 1 : Change-Point Detection

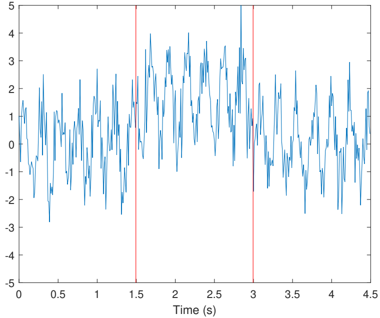


Change-Point Detection

Given a time series x, retrieve the times $(t_1, \ldots, t_K)$ where a significant change occurs

- Necessitates to estimate both the change-points but also the number of changes K
- Highly depends on the meaning given to change

Problem statement



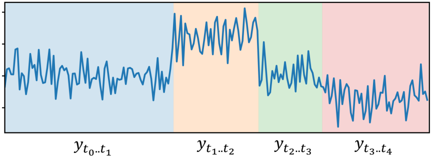
- When the changes are abrupt or when the estimation of the change-points is relevant in the context, we can use **change-point detection** methods
- Let assume that signal $x[n]$ undergoes abrupt changes at times

$$\mathcal{T}^* = (t_1^*, \ldots, t_{K^*}^*)$$

- Goal : retrieve the number of change-points and K^* and their times $\mathcal{T}^*$
- One assumption : offline segmentation (but can easily be adapted to online setting) [Truong et al., 2020]

Problem statement

$$(\hat{t}_1, \ldots, \hat{t}_K) = \underset{(t_1,\ldots,t_K)}{\text{argmin}} \sum_{k=0}^{K} c(x[t_k : t_{k+1}])$$

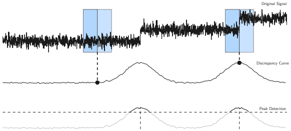


Cost function $c(.)$

- Measures the homogeneity of the segments
- Choosing $c(.)$ conditions the type of change-points that we want to detect
- Often based on a probabilistic model for the data

Problem solving

- Optimal resolution with dynamic programming
- Approximate resolution (sliding windows...)



Cost function

$$(\hat{t}_1, \ldots, \hat{t}_K) = \underset{(t_1, \ldots, t_K)}{\operatorname{argmin}} \sum_{k=0}^{K} c(x[t_k : t_{k+1}])$$

Convention : $t_0 = 0$, $t_{K+1} = N$

- Function $c(.)$ is characteristic of the notion of *homogeneity*
- The most common cost functions are linked to parametric probabilistic models : in this case change-points are defined as changes in the parameters of a probability density function [Basseville et al., 1993]
- Non parametric cost functions can also be introduced when no model is available

Maximum likelihood estimation

Given a parametric family of distribution densities $f(\cdot|\theta)$ parametrized with $\theta \in \Theta$, a cost function can be derived :

$$c_{ML}(x[a:b]) = -\sup_{\theta} \sum_{n=a+1}^{b} \log f\left(x[n]|\theta\right)$$

- Corresponds to the assumption that on a regime, samples are i.i.d. according to a parametric distribution density
- On each regime, the parameters are estimated through maximum likelihood estimation
- This model can be adapted to several situations : change in mean, change in variance, change in both mean and variance...

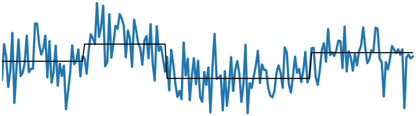
Change in mean

The most popular is indubitably the L2 norm [Page, 1955]

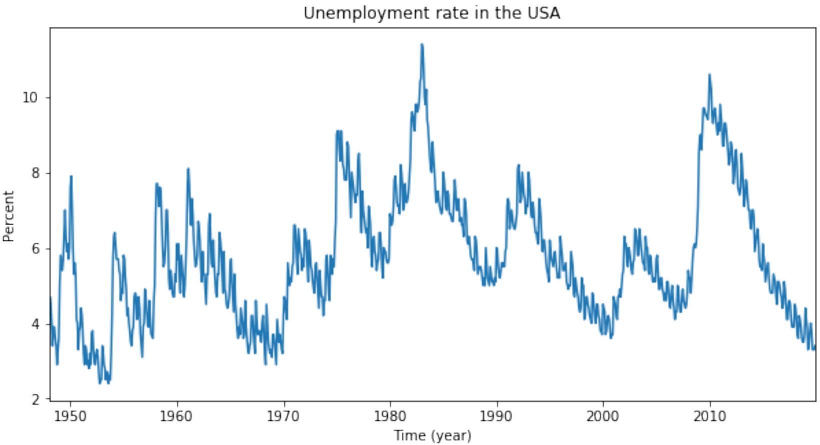
$$c_{L_2}(x[a:b]) = \sum_{n=a+1}^{b} \|x[n] - \mu_{a:b}\|_2^2$$

where $\mu_{a:b}$ is the empirical mean of the segment $x[a:b]$.

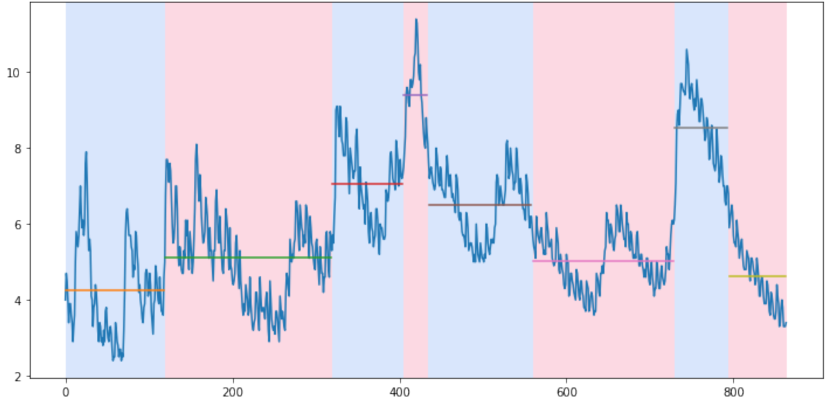
- Particular case of c_{ML} with Gaussian model with fixed variance
- Allows to detect changes in mean



Example

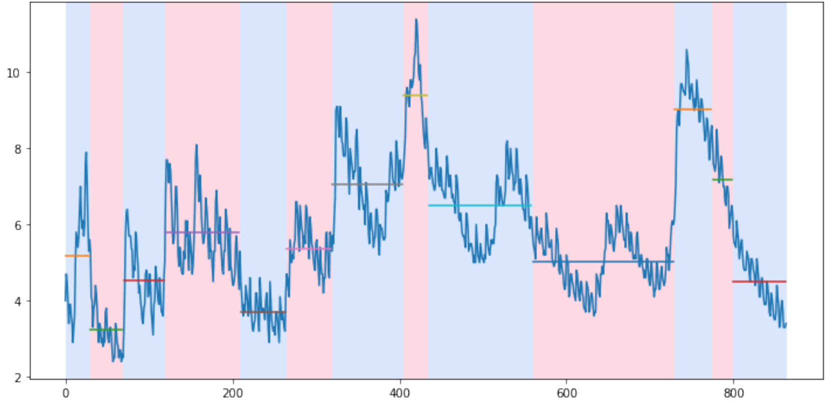

Unemployment rate in the USA
Percent
10
8
6
4
2
1950
1960
1970
1980
1990
2000
2010
Time (year)

Example : Change-Point Detection with c_{L_2}



$K = 7$

Example : Change-Point Detection with c_{L_2}



$K = 12$

Search method

$$(\hat{t}_1, \dots, \hat{t}_K) = \underset{(t_1, \dots, t_K)}{\operatorname{argmin}} \sum_{k=0}^{K} c(x[t_k : t_{k+1}])$$

Convention : $t_0 = 0$, $t_{K+1} = N$

- Several methods can be used to solve this problem with a fixed K
- Optimal resolution with dynamic programming : find the true solution of the problem (but costly)
- Approximated resolution with windows : test for one unique change-point on a window

Optimal resolution

- By denoting

$$\mathcal{V}(\mathcal{T}, \mathbf{x}) = \sum_{k=0}^{K} c(x[t_k : t_{k+1}])$$

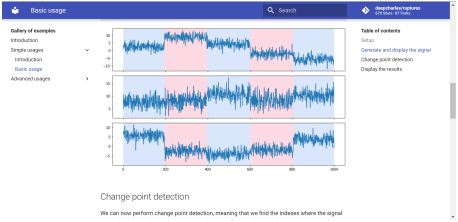
we can see that

$$\begin{aligned}
\min_{|\mathcal{T}|=K} V(\mathcal{T}, \mathbf{x}) &= \min_{0=t_0<t_1<\cdots<t_K<t_{K+1}=N} \sum_{k=0}^{K} c(x[t_k : t_{k+1}]) \\
&= \min_{t\leq T-K} \left[c(x[0 : t]) + \min_{t_0=t<t_1<\cdots<t_{K-1}<t_K=T} \sum_{k=0}^{K-1} c(x[t_k : t_{k+1}]) \right] \\
&= \min_{t\leq T-K} \left[c(x[0 : t]) + \min_{|\mathcal{T}|=K-1} V(\mathcal{T}, x[t : N]) \right]
\end{aligned}$$

- Recursive problem : resolution with dynamic programming [Bai et al., 2003]
- Two steps : computation of the cumulative costs + determination of the change-points

The ruptures package

ruptures



https://centre-borelli.github.io/ruptures-docs/

- Python library that contains most of the existing approaches for change-point detection
- Large variety of configurations (cost-functions) to solve a large choice of problems
- More than 11M downloads!

C. Truong, L. Oudre, and N. Vayatis. Selective review of offline change point detection methods. Signal Processing, 167 :107299, 2020

Supervised change-point detection

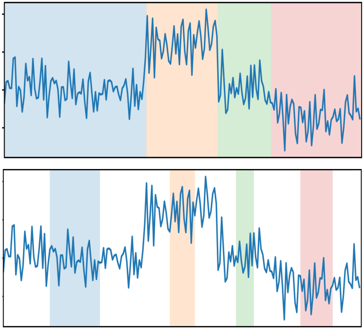
How to choose the number of breakpoints ?

- Supervised approches to learn, from a small set of annotated signals, the desired level of granularity for the detection process

How to choose the cost function ?

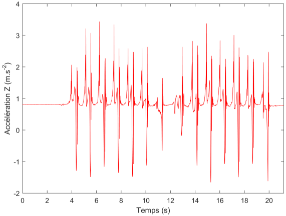
- Supervised approach to learn an adequate representation domain, where the change-points of interest are visible

- C. Truong and L. Oudre. Supervised change-point detection with dimension reduction, applied to physiological signals. In NeurIPS Workshop on Learning from Time Series for Health, 2022.
- C. Truong, L. Oudre, and N. Vayatis. Supervised kernel change point detection with partial annotations. In Proceedings of the International Conference on Acoustics, Speech, and Signal Processing (ICASSP), pages 3147-3151, Brighton, UK, 2019.
- C. Truong, L. Oudre, and N. Vayatis. Penalty Learning for Changepoint Detection. In Proceedings of the European Signal Processing Conference (EUSIPCO), pages 1614-1618, Kos Island, Greece, 2017.

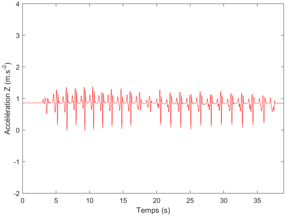


Complete or partial annotations

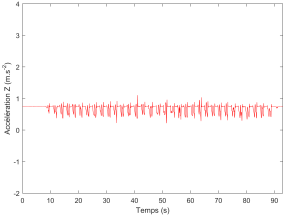
Back to the signals...



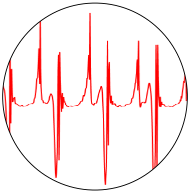
Healthy subject

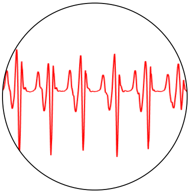


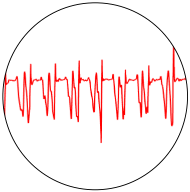
Mild neurological disease



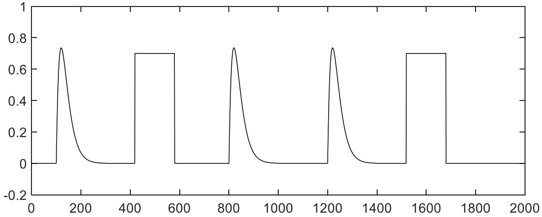
Severe neurological disease



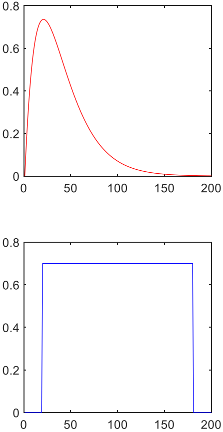




Problem 2 : Convolutional dictionary learning



Input time series



Extracted patterns

Problem 2 : Convolutional dictionary learning

Pattern Extraction

Given an input time series x (or a set of time series), learn a dictionary of patterns $\mathcal{P}$

- A template is a *shape* that appear repetitively in the time series (but kinda blurry notion)
- All templates are supposed to have the same length (for sake of simplicity)
- The extracted patterns can be used to characterize the time series, or studied individually

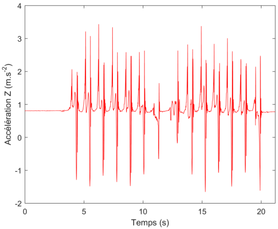
Dictionary-based pattern extraction

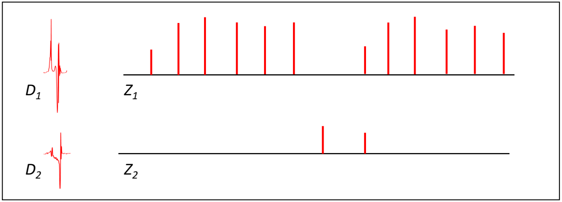
- Finding patterns in a time series can be seen as a dictionary learning optimization problem
- Given an input time series $\mathbf{x}$, learn a dictionary of K patterns $\mathbf{d}_k$ of length L
- These patterns can be activated : activations $\mathbf{z}_k$ of length $N - L + 1$

$$z_k[n] \neq 0 \text{ if pattern } \mathbf{d}_k \text{ is activated at time } n$$

[Grosse et al., 2007 ; Wohlberg, 2014]

Convolutional dictionary learning





Convolutional dictionary learning

Given a time series $\mathbf{x}$, number of pattern K and pattern length L, learn

- Patterns $\mathbf{d}_k$ of length L
- Activation signals $\mathbf{z}_k$ of length $N - L + 1$

$$x[n] = \sum_{k=1}^{K} (\mathbf{z}_k * \mathbf{d}_k)[n] + e[n]$$

Optimization problem

$$\min_{\substack{(\mathbf{d}_k),(\mathbf{z}_k)\\ \forall k, \|\mathbf{d}_k\|_2^2 \leq 1}} \left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|\mathbf{z}_k\|_1$$

- **Normalization constraint** for the dictionary atoms $\mathbf{d}_k$, that prevents numerical instabilities (otherwise setting $\alpha \mathbf{d}_k$ and $\alpha^{-1}\mathbf{z}_k$ gives the same result)
- **Sparsity constraint** for the activations $\mathbf{z}_k$, that improves the interpretability of the learned patterns

Not convex with respect to the couple $(\mathbf{d}_k), (\mathbf{z}_k)$ but convex when the subproblems are taken individually

Alternated resolution

Alternated resolution of two subproblems

Dictionary learning

$$\mathbf{D}^* = \underset{\substack{\mathbf{D}=(\mathbf{d}_1,\ldots,\mathbf{d}_K) \\ \forall k, \|\mathbf{d}_k\|_2^2 \leq 1}}{\operatorname{argmin}} \left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2$$

Convolutional sparse coding

$$\mathbf{Z}^* = \underset{\mathbf{Z}=(\mathbf{z}_1,\ldots,\mathbf{z}_K)}{\operatorname{argmin}} \left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|\mathbf{z}_k\|_1$$

Alternated resolution

$$\min_{\substack{(\mathbf{d}_k),(\mathbf{z}_k)\\ \forall k, \|\mathbf{d}_k\|_2^2 \leq 1}} \underbrace{\left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2}_{f(\mathbf{Z},\mathbf{D})} + \lambda \sum_{k=1}^{K} \|\mathbf{z}_k\|_1$$

- Both these problems can be solved with Proximal Gradient Descent algorithms
- Two main steps :
  1. Gradient descent step w.r.t. $\nabla_{\mathbf{D}} f(\mathbf{Z}, \mathbf{D})$ or $\nabla_{\mathbf{Z}} f(\mathbf{Z}, \mathbf{D})$
  2. Proximal step to *project* the update on the constraint set

Dictionary learning

$$\mathbf{D}^* = \underset{\substack{\mathbf{D}=(\mathbf{d}_1,\ldots,\mathbf{d}_K)\\ \forall k, \|\mathbf{d}_k\|_2^2 \leq 1}}{\operatorname{argmin}} \underbrace{\left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2}_{f(\mathbf{Z},\mathbf{D})}$$

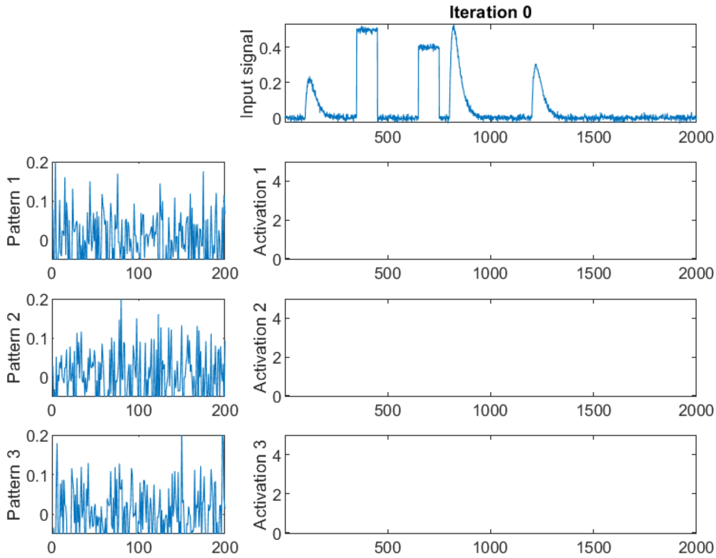
- Basic approach : **Proximal Gradient Descent** with fixed $\mathbf{Z}$
  1. Gradient step :
  $$\mathbf{D} \leftarrow \mathbf{D} - \alpha \nabla_{\mathbf{D}} f(\mathbf{Z}, \mathbf{D})$$
  2. Proximal projection step :
  $$\mathbf{d}_k \leftarrow \operatorname{proj}_{\|.\|_2^2 \leq 1}(\mathbf{d}_k) = \frac{\mathbf{d}_k}{\max(\|\mathbf{d}_k\|_2^2, 1)}$$
- Other approaches : Alternate Direction Method of Multiplier (ADMM), K-SVD... (see [Mairal et al., 2010])

Convolutional sparse coding

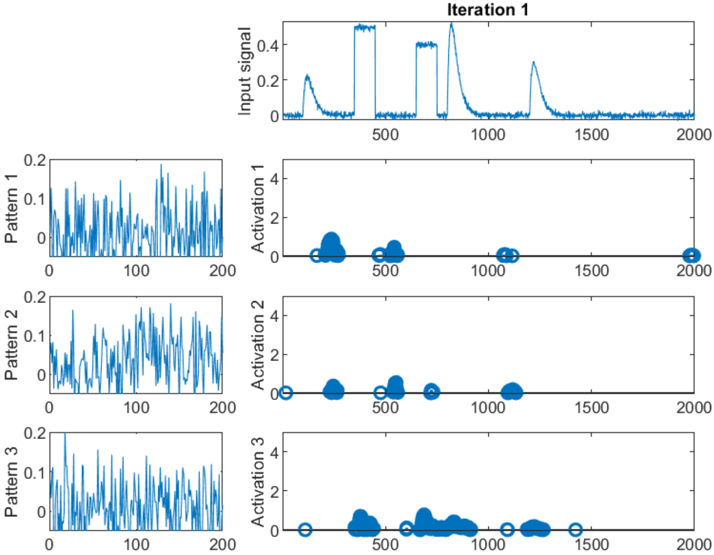
$$\mathbf{Z}^* = \underset{\mathbf{Z}=(\mathbf{z}_1,\dots,\mathbf{z}_K)}{\operatorname{argmin}} \underbrace{\left\| \mathbf{x} - \sum_{k=1}^{K} \mathbf{z}_k * \mathbf{d}_k \right\|_2^2}_{f(\mathbf{Z},\mathbf{D})} + \lambda \underbrace{\sum_{k=1}^{K} \|\mathbf{z}_k\|_1}_{\psi(\mathbf{Z})}$$

- Basic approach : **Iterative Soft Thresholding Algorithm (ISTA)** with fixed $\mathbf{D}$ [Daubechies et al., 2004]
  1. Gradient step
  $$\mathbf{Z} \leftarrow \mathbf{Z} - \alpha \nabla_{\mathbf{Z}} f(\mathbf{Z}, \mathbf{D})$$
  2. Proximal step
  $$\mathbf{Z} = \mathcal{S}_{\lambda\alpha}(\mathbf{Z})$$
- Other approaches : Alternate Direction Method of Multiplier (ADMM), Fast Iterative Soft Thresholding Algorithm (FISTA), Coordinate Descent (CD) (see [Mairal et al., 2010])

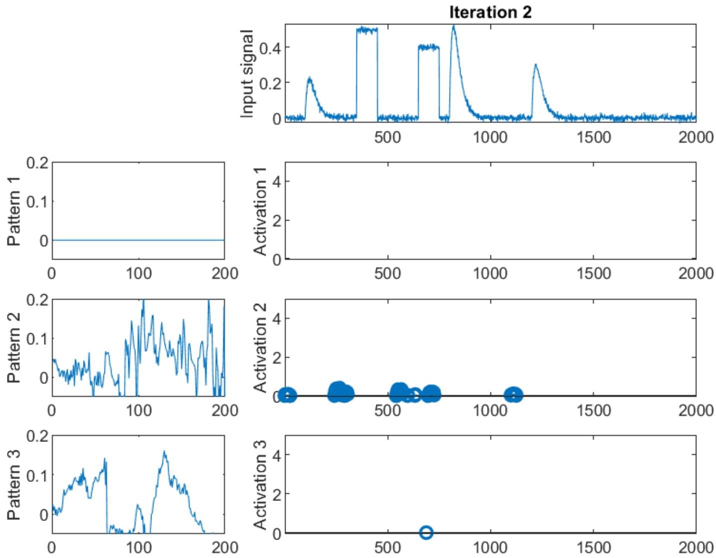
Results


Iteration 0
Input signal
0.4
0.2
0
500
1000
1500
2000
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3
0.2
0.1
0
100
200
4
2

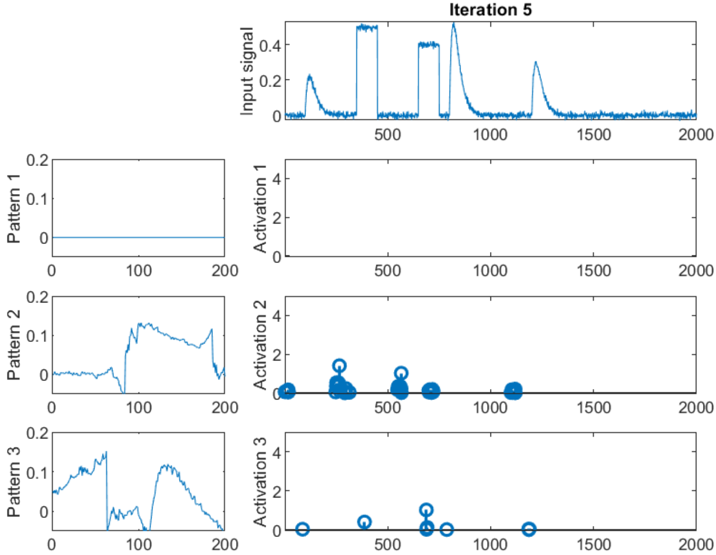
Results


Iteration 1
Input signal
0.4
0.2
0
500
1000
1500
2000
Pattern 1
Pattern 2
Pattern 3
0.2
0.1
0
0
100
200
Activation 1
Activation 2
Activation 3
4
2
0

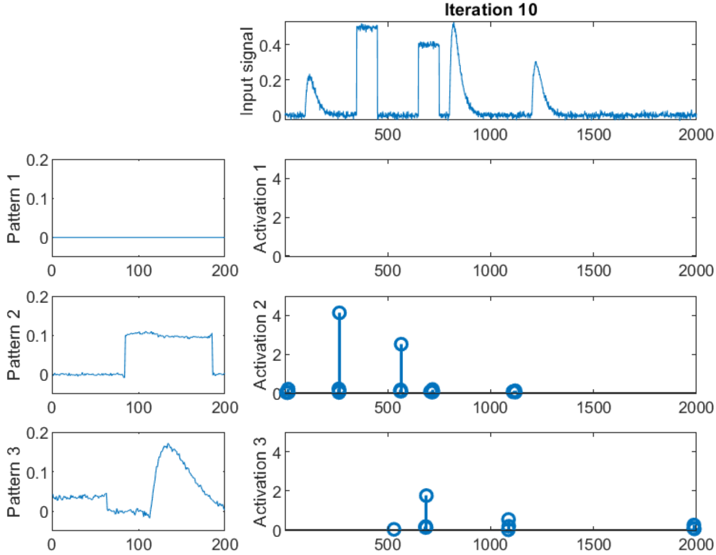
Results


Iteration 2
Input signal
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3

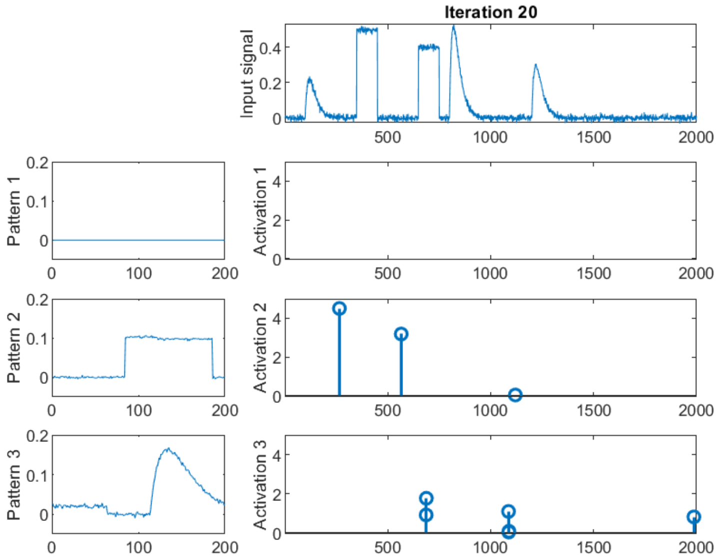
Results


Iteration 5
Input signal
0.4
0.2
0
500
1000
1500
2000
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3
0.2
0.1
0
0
100
200
4
2

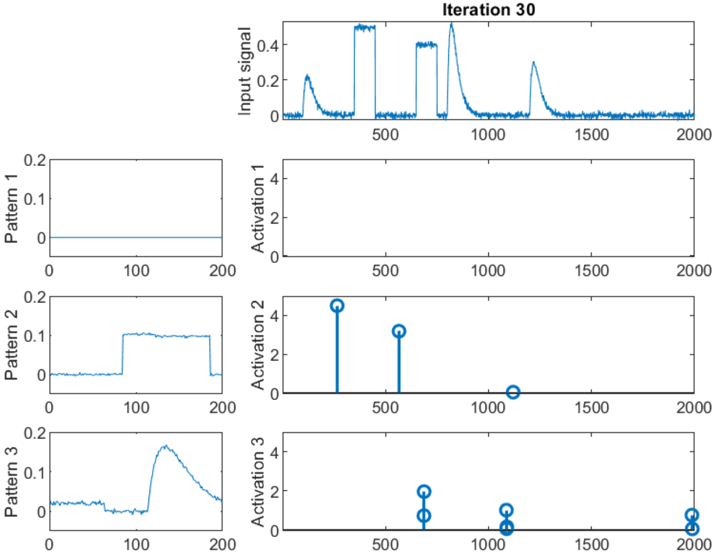
Results


Iteration 10
Input signal
0.4
0.2
0
500
1000
1500
2000
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3
0.2
0.1
0
100
200
4
2

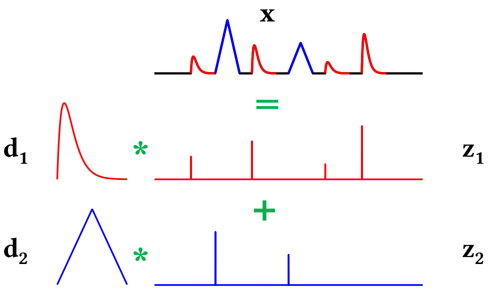
Results


Iteration 20
Input signal
0.4
0.2
0
500
1000
1500
2000
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3
0.2
0.1
0
0
100
200
4
2

Results


Iteration 30
Input signal
Pattern 1
Activation 1
Pattern 2
Activation 2
Pattern 3
Activation 3

Convolutional dictionary learning



$$x[t] = \sum_{k=1}^{K} (z_k * d_k)[t] + e[t]$$

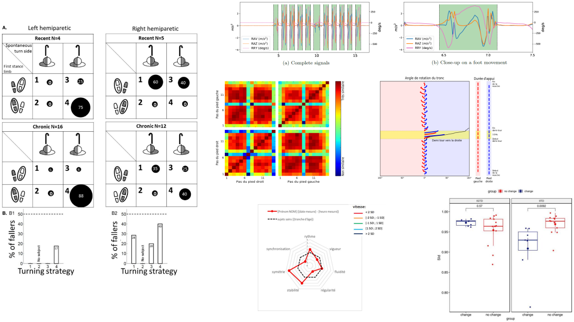
- Parallelized approaches for processing long signals (DICOD)

T. Moreau, L. Oudre, and N. Vayatis. DICOD : Distributed Convolutional Coordinate Descent for Convolutional Sparse Coding. In Proceedings of the International Conference on Machine Learning (ICML), pages 3626-3634, Stockholm, Sweden, 2018.

PART III

Perspectives

Use for biomedical research



- Study of U-turn for post-stroke patients [Barrois-Müller et al., 2017]
- Step analysis for multiple sclerosis patients [Vienne-Jumeau et al., 2020]
- Comparison of gait exercises through pattern matching techniques [Vienne-Jumeau et al., 2019]

Other projects

Many other studies in progress :

- **Study of human posture :** Romberg test on force platforms (2D trajectories) to detect the risk of falling in seniors
- **States of consciousness :** Recording of numerous physiological variables (EEG, ECG, respiratory and blood concentration variables) in order to better understand the phases of sleep/wake up and to give a quantification of the depth of anesthesia
- **Upper limb analysis :** Quantification and assessment of the impact of rehabilitation using accelerometers
- **Oculometry :** Measurement of eye movement in infants for the follow-up of pathologies such as Nystagmus
- **Behavior and mental workload :** Connected apartment for the study of neurological pathologies, study of mental workload for plane/helicopter/train pilots/drivers, study of the behavior of motorists in pre-accident situations...

Thank you for your attention